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Quantum Information and Computation Initiative

Quantum operations with indefinite time direction

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The fundamental dynamics of quantum particles is neutral with respect to the arrow of time. Agents such as us observe quantum systems evolving from the past to the future. In principle, some other type of agent could perform experiments in the opposite direction and even in a superposition of the two directions. In this work, we establish a framework for describing the interactions of an agent with the fundamentally time-symmetric dynamics of quantum theory, and for composing local processes into more complex structures:

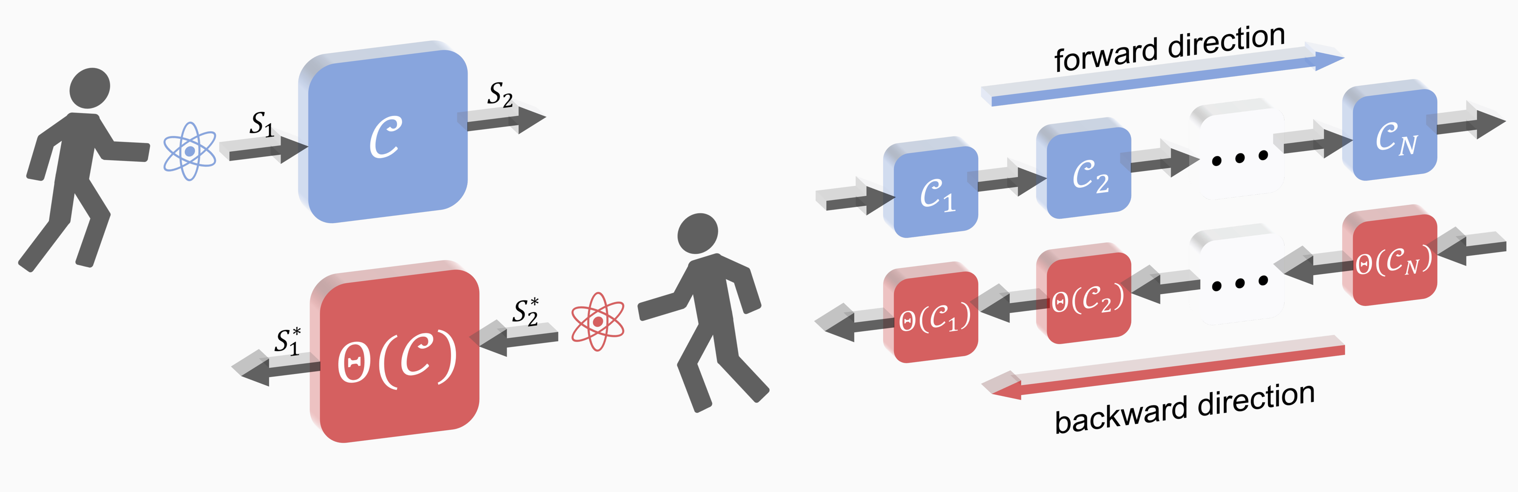
- A mathematical framework for operations that are not constrained to a definite time direction;
- A set of multipartite operations that include indefinite time direction as well as indefinite causal order, providing a framework for potential extensions of quantum theory;
- An information theoretic advantage of quantum operations with indefinite time direction.

Bidirectional devices and their characterisation

To determine whether a given process is bidirectional, one has to specify a map Θ, converting the channel $\mathcal{C}$ observed by the forward-facing agent into the corresponding channel $\Theta(\mathcal{C})$ observed by the backward-facing agent.

We characterize the set of bidirectional devices based on four natural requirements for input-output inversion:

1. *order-reversing:* $\Theta(\mathcal{D}\,\mathcal{C}) = \Theta(\mathcal{C})\,\Theta(\mathcal{D})$ for every pair of channels $\mathcal{C}$ and $\mathcal{D}$,
2. *identity-preserving*: $\Theta(\mathcal{I}_S) = \mathcal{I}_{S^*}$, where $\mathcal{I}_S$ (S^*) is the identity channel on system S (S^*).
3. *distinctness-preserving:* if $\mathcal{C} \neq \mathcal{D}$, then $\Theta(\mathcal{C}) \neq \Theta(\mathcal{D})$,
4. *compatible with random mixtures*: $\Theta\,(p\,\mathcal{C} + (1-p)\,\mathcal{D}) = p\,\Theta(\mathcal{C}) + (1-p)\,\Theta(\mathcal{D})$ for every pair of channels $\mathcal{C}$ and $\mathcal{D}$, and for every probability $p \in [0,1]$.



- The set of bidirectional processes coincides with the set of bistochastic channels [1, 2];
- Up to unitary equivalence, there exist only two possible choices of input-output inversion: the adjoint $\mathcal{C} \mapsto \mathcal{C}^\dagger$ and the transpose $\mathcal{C} \mapsto \mathcal{C}^T$.

Quantum operations with indefinite time direction

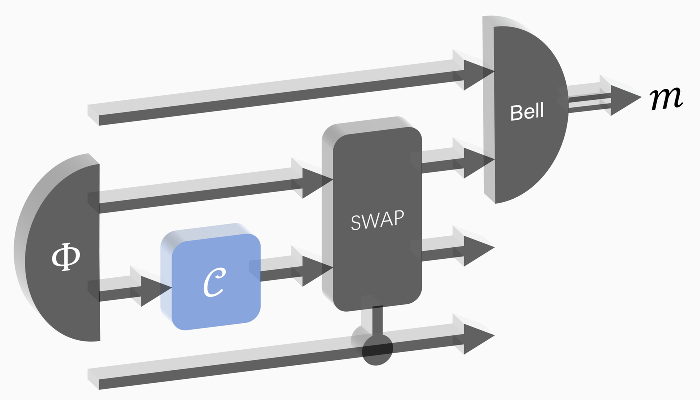
The possible operations on bidirectional devices correspond to quantum supermaps transforming bistochastic channels into ordinary channels (CPTP maps)

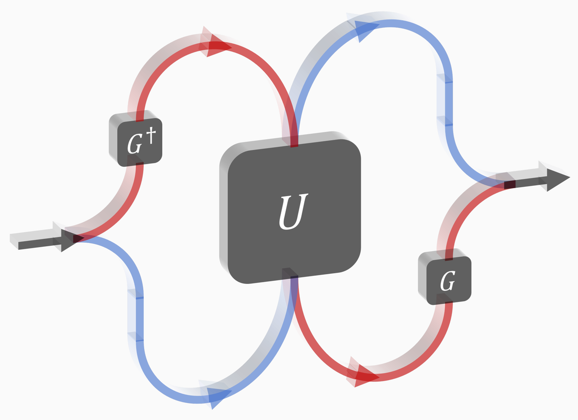
- A forward supermap can be realized in the form $\mathcal{S}_{\text{fwd}}(\mathcal{C}) = \mathcal{B}(\mathcal{C} \otimes \mathcal{I}_{\text{aux}})\mathcal{A}$;
- a backward supermap can be realized in the form $\mathcal{S}_{\text{bwd}}(\mathcal{C}) = \mathcal{B}'(\Theta(\mathcal{C}) \otimes \mathcal{I}_{\text{aux}'})\mathcal{A}'$;
- supermaps with indefinite time direction can not be realized by a random mixture of a forward supermap and a backward supermap.

The quantum time flip is a concrete example of an operation with indefinite time direction, which creates a coherent superposition of a process with its input-output inverse. Let $\{C_k\}$ be a Kraus representation of a bistochastic channel $\mathcal{C}$, and let $\mathcal{F}$ denote the quantum time flip supermap. Then the Kraus representation of $\mathcal{F}(\mathcal{C})$ is

$$F_k = C_k \otimes |0\rangle\langle 0| + C_k^T \otimes |1\rangle\langle 1| \,. \tag{1}$$

The quantum time flip cannot be perfectly realized by any quantum circuit with a definite time direction. In a teleportation setup, the quantum time flip can be realized probabilistically. We can also implement the quantum time flip in a photonic setup.





Higher order quantum operations corresponds to **quantum supermaps** [3, 4, 5, 6] from an input set of quantum channels B to an output set of quantum channels B'. A quantum supermap

- preserves convex combinations, and
- can act locally on the dynamics of composite systems.

An information-theoretic advantage

Given two black boxes which implement two unitary gates U and V satisfying either

- the condition $UV^T = U^T V$, or
- the condition $UV^T = -U^T V$

The goal of the player is to discover which of these two alternatives holds.

- A player with access to the quantum time flip can win the game with certainty.
- Every player who can only probe the two unknown gates in a definite time direction will have a probability of at least 11% to lose the game.

In the winning strategy, one time flip generates the gate $S_U = U \otimes |0\rangle\langle 0| + U^T \otimes |1\rangle\langle 1|$, while the other generates the gate $S_V = V^T \otimes |0\rangle\langle 0| + V \otimes |1\rangle\langle 1|$. The player can then obtain the state

$$\begin{aligned} S_U S_V (|\psi\rangle \otimes |+\rangle) = & \left[\frac{UV^T + U^T V}{2}|\psi\rangle\right] \otimes |+\rangle \\ & + \left[\frac{UV^T - U^T V}{2}|\psi\rangle\right] \otimes |-\rangle \,. \end{aligned} \tag{2}$$

and measure the control qubit in the basis $\{|+\rangle, |-\rangle\}$ to figure out exactly which condition is satisfied.

Multipartite quantum operations on bistochastic channels

Multipartite operations with indefinite time direction can also be described as quantum supermaps on the set of N-partite no-signalling bistochastic channels.

Three bipartite examples (let $\{A_{1m}\}$ be the Kraus representation of $\mathcal{A}_1$ and $\{A_{2n}\}$ be the Kraus representation of $\mathcal{A}_2$):

$$\mathcal{S}^{(i)}(\mathcal{A}_1 \otimes \mathcal{A}_2)(\rho) := \sum_{m,n} S_{m,n}^{(i)}\, \rho\, S_{m,n}^{(i)\,\dagger}$$

1. indefinite time direction & definite causal order
$$S_{m,n}^{(1)} = A_{1m} A_{2n}^T \otimes |0\rangle\langle 0| + A_{1m}^T A_{2n} \otimes |1\rangle\langle 1| \,;$$
2. definite time direction & indefinite causal order (quantum SWITCH)
$$S_{m,n}^{(2)} = A_{1m} A_{2n} \otimes |0\rangle\langle 0| + A_{2n} A_{1m} \otimes |1\rangle\langle 1| \,;$$
3. indefinite time direction & indefinite causal order
$$S_{m,n}^{(3)} = A_{1m} A_{2n} \otimes |0\rangle\langle 0| + A_{2n}^T A_{1m}^T \otimes |1\rangle\langle 1| \,.$$

Applications of the framework

- Experimental demonstration of quantum operations with input-output indefiniteness [7, 8];
- Quantum communication through a single channel in an indefinite input-output direction [9].

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